The Intra-day Market Price of Money: evidence from e-mid market*

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Abstract

We present a simple model, where intraday and overnight interest rates are linked by a no-arbitrage argument. The hourly interest rate is shown to be a function of the intraday term structure of the overnight rate. This property holds under both assumptions, where an explicit intraday market for interbank loans exists and when it does not. In the first case, such a property is an equilibrium condition; in the second one it holds by definition, as a synthetic hourly loan is a portfolio of overnight contracts. We then provide empirical evidence, based on tick-by-tick data for the e-MID euro-area money market (covering 2003 and 2004). The overnight rate shows a clear downward pattern throughout the operating day. A positive hourly interest rate emerges from the intraday term structure of the overnight rate: we estimate the market price of a one hour interbank loan to be slightly below a half basis point.

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1 Introduction

Is there a market price for "intraday money"? The answer to this question is interesting for banks, as it gives some insights for their liquidity management and it helps understanding the working of payment systems, and it is relevant for central banking, in particular for grasping the implications of different policies in the provision of intraday credit. Actually, an explicit market for loans with shorter maturities than one day does not (yet) exist. The shortest maturity for which an interest rate is quoted in financial markets is everywhere the overnight. The rate prevailing in the overnight interbank market is also the operational target of monetary policy in several countries (including USA and the euro area). However, an implicit price for intraday transactions does exist - despite the fact that such transactions do not actually take place - whenever the overnight interest rate differs, depending on the exact time (within the same day) at which the delivery/repayment of funds takes place. So for example, the difference (if any) between the rate charged on an overnight loan delivered at 9 a.m. and a loan with the same maturity delivered at 10 a.m. implicitly defines the price of a one hour loan.

The reasons behind a positive intraday interest rate essentially rely on the organization of payment systems and, more generally, on the way in which the settlement of transactions is handled. When the bulk of payments were settled through netting systems, a sort of "free intraday liquidity" was provided by the netting mechanism itself: only the multilateral balance of payments had to be settled at the end of the day. During the nineties, the real time gross settlement (RTGS) has become widely used, particularly for large value payments. This method of handling payments is highly demanding: banks have to maintain sufficient (idle) balances with the central bank, to be able to settle payments one by one in real time; alternatively, they may rely on central bank intraday credit, which also comes at some cost. The real time settlement has become the common standard also for securities transactions, with delivery versus payment (DVP). Finally, a payment-versus-payment (PVP) approach has been adopted by CLS (Continuous Linked Settlement), dealing with foreign exchange transactions.

More recently, the trend towards "hybrid" systems - implementing a sort of real time *net* settlement - has somehow reduced the liquidity needed for settlement purposes. However, these systems also create an incentive for banks to actively manage their liquidity during the operating day (for example in order to optimally allocate the available liquidity to different categories of payments and to efficiently manage payment queues). A role for an active intraday liquidity management is also created by several cut-off times to be met by bank treasury departments during the day (think for example of "timed payments" scheduled by CLS).[1]

On theoretical grounds, the emergence of an intraday interest rate in the interbank market has been advocated by VanHoose (1991) and Angelini (1998): they both model bank liquidity management at an intraday level, distinguishing between a "morning session" and an "afternoon session". Despite some differences (the former focusses on trading in the interbank market, while the latter focusses on the timing of payment orders processing), they reach the same basic results: i) a positive value of the intraday interest rate emerges as the equilibrium level in the interbank market; ii) such a level crucially depends on the price of daylight overdrafts charged by the central bank; iii) absent an explicit market for intraday transactions, the intraday interest rate is computed as the difference between the overnight rate on interbank loans delivered in the morning and the rate on loans delivered in the afternoon, providing an implicit price for money between the two periods within the same day. The strategic choice of banks relative to the timing of payment sending is also the core of the analysis done by Beck and Garratt (2003), who assume that the interest rate in the (implicit) intraday money market equals the cost of intraday liquidity supplied by the central

[1] Examples of hybrid systems are: RTGS-plus, PNS, New BI-Rel in Europe, and CHIPS in USA. For a more detailed analysis of recent changes in payment systems and their implications for central banking, see Baglioni (2006).

bank: this cost either takes the form of an explicit fee or it is the opportunity cost of pledging collateral. Finally, Zhou (2000) stresses the distinction between "settlement debt" (intraday) and "consumption/investment debt" (across days): only the latter affects the intertemporal allocation of resources, while the former arises for pure settlement purposes; under this approach a "day" may be defined as any length of time over which there is no point in optimizing the timing of consumption and production.

The empirical evidence regarding the price of intraday liquidity is still modest and not conclusive. As far as we know, the only analysis pointing to the existence of an implicit market for intraday interbank lending has been done by Furfine (2001): he finds that in the federal funds market an additional hour is priced 0.9 basis point; this result is attributed to the cost of borrowing from the central bank through the daylight overdraft facility[2]. Angelini (2000) does not find any clear intraday pattern of the overnight rate prevailing in the Italian screen-based interbank market (MID, 1993-1996 data): in particular, such rate turns out to be slightly (less than two basis points) below its midday level both in the early morning and in the early afternoon.

The aim of this paper is twofold. First, we want to make clear the theoretical relationship between intraday and overnight interest rates. As we are going to see, they are linked by a no-arbitrage argument: the hourly interest rate is determined by the intraday term structure of the overnight rate, where the latter is defined by the levels of the overnight rate for different durations of the contract. This property applies to both cases, where an explicit intraday market for interbank loans does exist and when it does not (in the

[2]The Fed charges an annualized daily rate of 36 basis points; this amounts to a 1.5 b.p. hourly fee. The effective daily rate is 27 b.p. (36 b.p. times 18/24, as Fedwire operating hours are 18) divided by 360. The daily charge (neglecting the deductible) results from the product between such a rate and the average per-minute overdraft incurred by a bank in a day (see McAndrews and Rajan (2000) for more details). Therefore, repaying a 1 dollar federal fund loan one hour later enables a bank saving 1.5 basis points (annualized), given that it is running an overdraft at the time of repayment.

latter case, synthetic intraday loans are created through overnight contracts delivered at different times during the day). Second, we provide some empirical evidence, based on tick-by-tick data for the e-MID euro-area money market, showing that a positive hourly interest rate (implicitly) emerges in the overnight market: we estimate the market price of a one hour interbank loan to be slightly below a half basis point.

2 An arbitrage model for the intraday interest rate

In this section we lay out a simple model, where intraday and overnight interest rates are linked by a no-arbitrage argument. We focus on a single day, and denote by $t = 0, 1, ..., 24$ the hours during the day: $t = 0$ denotes the opening time of the interbank market (say 9 a.m.), $t = 1$ one hour later and so forth until $t = 24$ (9 a.m. of the next day). The interbank market closing time is T: in principle, it might be $T = 24$; in practice, it is $T < 24$; in the following, we will consider different cases about the value of T. Assume that all overnight interbank loans have to be repaid at $t = 24$. We call r_0 the interest rate on an overnight interbank loan delivered at $t = 0$; r_1 is the rate on a loan delivered at $t = 1$, and so forth until r_T. So the list $[r_0, r_1, ..., r_T]$ describes the "intraday term structure" as the levels of the overnight rate for different durations: 24 hours, 23 hours, ..., $24 - T$ hours respectively.

2.1 An explicit market for hourly interbank loans

Assume that in the interbank market it is possible to trade on an hourly basis: for example, a bank may borrow funds from $t = 2$ to $t = 3$. This assumption is not realistic, and we are going to drop it in the next subsection, but it is useful to begin our analysis; on theoretical grounds, it is equivalent to assuming that there is no transaction cost. Assume further, for simplicity, that the hourly interest rate is constant throughout a single day, and it is denoted by $r \geq 0$ (we have discussed in the Introduction the reasons why r might be strictly positive).

It is easy to see that the following no-arbitrage condition must hold in equilibrium:

$$(1+r)^t(1+r_t) = 1 + r_0 \qquad \text{for } t = 1, ..., T \tag{1}$$

In words, a roll-over strategy of investing in hourly contracts for t hours (starting at $t = 0$) and in an overnight contract for the remaining $24 - t$ hours must provide the same return as a 24-hour length overnight contract. Otherwise, arbitrage opportunities would arise. Condition (1) holds for both $r > 0$ and $r = 0$. In the latter case the intraday term structure is flat ($r_t = r_0$ for $t = 1, ..., T$), while in the former case a decreasing intraday term structure emerges ($r_0 > r_1 > ... > r_T$).

By taking the log of condition (1), we can easily derive the following:

$$r = \frac{r_0 - r_t}{t} \qquad \text{for } t = 1, ..., T \tag{2}$$

where the equilibrium (arbitrage free) hourly interest rate is a function of the intraday term structure of the overnight rate.

From (2) it is evident that a specific level of the overnight rate is compatible with any level of the hourly rate: the latter depends only on the *difference* between overnight rates with different durations, so the *level* of r_0 is irrelevant for determining r.

This property fails to hold when $T \geq 23$ (this is not a realistic case, and we consider it only for completeness). It is intuitive to set $r_{23} = r$ and $r_{24} = 0$. Then for $t = 23, 24$ conditions (1) and (2) respectively become:

$$(1+r)^{24} = 1 + r_0 \quad \text{and} \quad r = \frac{r_0}{24} \tag{3}$$

implying that the level of the 24-hour overnight rate uniquely determines the level of the hourly rate: in fact, the latter is simply a fraction of the former. This simple relation would hold if it were possible to replicate an overnight contract with 24 hourly contracts. This is not true in practice.

2.2 An implicit market for hourly interbank loans

We come now to the more realistic assumption that transaction costs prevent an explicit market for hourly interbank loans to arise. However, synthetic hourly contracts may be created by making use of overnight loans with different delivery times. Then the hourly interest rate turns out to be implicitly defined by the intraday term structure of the overnight rate. Take for example the following position in the overnight market: lend at $t = 0$ and borrow at $t = 1$; this is equivalent to lending for one hour and it gives a return equal to $r_0 - r_1$. More generally, a synthetic long position in the interbank market for t hours may be created by lending overnight at $t = 0$ and borrowing the same amount at t (with $1 \leq t \leq T$); the hourly return on such a position is defined as in equation (2).[3]

Therefore, when the hourly interest rate is implicitly defined by the intraday term structure of the overnight rate - as it is the case when no explicit hourly market exists - the no-arbitrage condition (1) is trivially satisfied. This is not surprising, as r is defined as an implicit price in the overnight market, rather than being the equilibrium price of an explicit hourly market[4].

Absent an explicit hourly market, the hourly interest rate r is not observable. However, equation (2) provides a way to estimate such rate by exploiting the intraday term structure in the overnight market. Let us write that equation as:

[3] Of course, a short position for t hours and its hourly cost are defined in a similar way.

[4] Another way to look at this point is by considering a roll-over strategy of investing in a synthetic long position for t hours (starting at $t = 0$) and lending overnight for the remaining $24-t$ hours: this boils down to lending overnight with a 24-hour length contract, giving a return equal to r_0; again, condition (1) is trivially met.

$$r_t = r_0 - r \cdot t \tag{4}$$

where the overnight rate linearly depends on the time of delivery. The estimated coefficient of this regression line provides an empirical measure of the hourly interest rate. This task is taken up in the next section.

3 Empirical analysis

An empirical analysis was performed using data from the e-MID money market to investigate the existence of an implicit intraday market for bank liquidity. E-MID is a screen-based market located in Milan and it is currently the most liquid market in the euro zone for the exchange of interbank deposits. This market has expanded considerably in recent years and it is now fully used by major European banks: indeed, non Italian banks account for about 40% of daily trades (as of March 2005).

Dealings in e-MID start at 8 a.m. and end up at 6 p.m. One important aspect of the market microstructure concerns the overnight contract: this has a fixed maturity time. In particular, dealings between Italian banks matures at 9 a.m. of the day following the one in which the contract was made: at this time previous day trades are settled in real time, as the borrowing bank has to repay the amount due through a Target payment. Dealings involving - at least - a foreign bank mature by noon (next day). This feature enables us to apply the framework introduced in the previous section, where the starting time of a contract unambiguously determines the length of such a contract, and this is known by both participants in the deal.

For the purposes of our analysis we consider trades (tick-by-tick data) that occurred in the e-MID market from 2003:01:02 to 2004:12:31, for a total of 293,667 observations[5]. In order to measure changes in the overnight rate as a function of different times of the day, we divided the day into 9 hourly

[5] We thank e-MID s.p.a. for providing this data set to us.

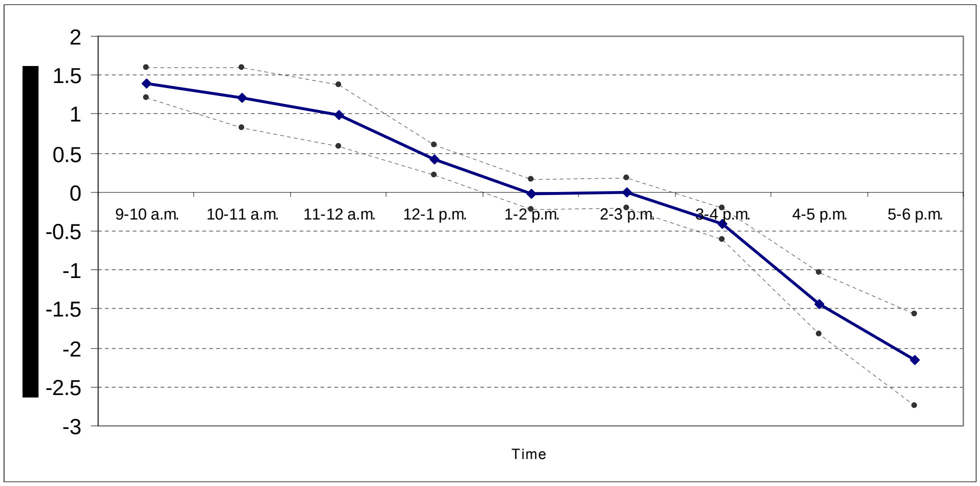

Figure 1: The intraday pattern of interest rates

time bands from 9 a.m. to 6 p.m., denoted by $t = 0, ..., 8$. The period between 8.00 a.m. and 9.00 a.m. was not considered because there are too few data. In fact, since in most of the days in this time band there are less than 10 observations[6], the significance is wanting. For each working day the simple average daily overnight rate, the average overnight rate for each hourly time band and the differences between the latter and the former were calculated: these differences will be denoted by $\overline{r}_t$ in the following. By making use of interest rate differentials from the daily average, instead of relying on their levels, we are able to insulate intraday patterns - which are our focus - from day-to-day changes in money market rates. Monthly aggregates were then computed: the regression below was then run on 4617 (9 times bands times 513 days).

It is then possible to see whether intraday changes in overnight rates are linked to the different hourly time bands. Before coming to the econometric analysis, a quick look at the data is quite suggestive: see Figure 1, where the

[6] While the average number of the observations for each time band is roughly equal to 64.

differentials between the overnight rates in each time band and the daily average are plotted (data are aggregated over the whole period). The intraday pattern of the overnight rate is clearly shown in the picture, with a steady decline over the whole day - taking a break during lunch time. The overall decrease amounts to almost 3.5 basis points.

The impression given by the picture is fully confirmed by the econometric analysis conducted on the sample data: we estimate the parameters of equation (5). This assumes that the overnight interest rate is a linear function of the time when a trade takes place:

$$\overline{r}_t = c + \sum_{i=1}^{8} \beta_i x_i + \varepsilon_t \tag{5}$$

where c is the constant and ε_t is the usual white noise. The x_i are dummy variables - where i stands for the hourly time bands following the first one - taking value 1 when $t = i$ and zero otherwise. The regression results[7] are given in Table 1. The intercept provides an estimate for the deviation of the overnight rate in the first hourly band considered (9-10 a.m.) from the daily average. The value of each β_i provides an estimate of the change of the overnight rate between the beginning of the day and the hourly band $t = i$. As p-values show, all coefficients are significant at 5% level (with the only exception of β_1).

[7] The estimates used in this paper come from Time Series Modelling Version 4; see Davidson (2006).

Table 1 - Estimated regression

Coefficient	Value	p-value	$\beta_i - \beta_{i-1}$
c	1.40	0	
β_1	-0.19	0.35	-0.19
β_2	-0.42	0.03	-0.23
β_3	-0.99	0	-0.57
β_4	-1.43	0	-0.44
β_5	-1.41	0	0.02
β_6	-1.81	0	-0.40
β_7	-2.83	0	-1.02
β_8	-3.55	0	-0.72
R^2	0.09		
			$Mean = -0.44$

It can be seen from the table that the overnight rate in the first hourly band is almost 1.4 basis points higher than the daily average. As the beta values show, the overnight rate gradually declines during the day; in the last operating hour, its level is almost 3.5 basis points below the initial level. The last column ($\beta_i - \beta_{i-1}$) shows that the change of the overnight rate with respect to the previous hour is negative in all time bands, with only one exception (namely between the 1-2 p.m. and the 2-3 p.m. bands).

The above evidence seems to confirm the hypothesis that an implicit intraday money market exists: the price of an overnight interbank loan does depend on the length of the contract; in other words, the intraday term structure of the overnight interest rate defines a strictly positive hourly rate in the money market.

Equation (5), used for regression purposes, is an extension of equation (4): the latter relies on the simplifying assumption that the hourly interest rate is constant throughout the operating day. A synthetic measure of the hourly interest rate charged in the money market is obtained by restating our results as in equation (6) below, by making use of the beta estimated values. This provides an empirical evaluation of equation (4) above, and it shows that the hourly price of money is estimated to be 0.44 bp on average.

$$\overline{r}_t = 1.4 - 0.44t \tag{6}$$

4 Summary and conclusions

We have presented a simple model, where intraday and overnight interest rates are linked by a no-arbitrage argument: even when it is not possible to replicate an overnight contract with intraday contracts, a relationship between hourly and overnight interest rates must hold to avoid arbitrage opportunities. Such a relationship makes the hourly rate be a function of the intraday term structure of the overnight rate. This property holds under both assumptions, where an explicit intraday market for interbank loans exists and when it does not. In the first case, such a property is an equilibrium condition; in the second one it holds by definition, as a synthetic hourly loan is a portfolio of overnight contracts.

We then provide empirical evidence, based on tick-by-tick data for the e-MID money market (covering the years 2003 and 2004). The overnight rate shows a clear downward pattern throughout the operating day. Therefore, a positive hourly interest rate emerges from the intraday term structure of the overnight rate: we estimate the market price of a one hour interbank loan to be slightly below a half basis point.

Compared with previous results relative to the MID market (Angelini 2000), our estimates point to an evolution of the interbank market: a price for intraday loans did not emerge during the mid-nineties, while it does a few years later. This pattern confirms what we presumed in our Introduction, namely that the recent evolution of settlement procedures calls for a more active intraday management of bank liquidity.

On the other hand, our estimate for a one hour interest rate is lower than that (0.9 basis point) obtained by Furfine (2001) for the federal funds market. The reason may be found in the absence of an explicit fee for the intraday liquidity provided by the ECB, contrary to what happens in US. In the euro area, the cost of the central bank intraday credit is only an implicit one, namely the opportunity cost of pledging collateral. Actually, our estimate for the intraday interbank interest rate might be interpreted as a market price of collateral.

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